

A Model of Low-lying States in Strongly Interacting Electroweak Symmetry-Breaking Sector

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Abstract

It is proposed that, in a strongly-interacting electroweak sector, besides the Goldstone bosons, the coexistence of a scalar state (H) and vector resonances such as A_1 [$I^G(J^P) = 1^-(1^+)$], V [$1^+(1^-)$] and ω_H [$0^-(1^-)$] is required by the proper Regge behavior of the forward scattering amplitudes. This is a consequence of the following well-motivated assumptions: (a). Adler-Weisberger-type sum rules and the superconvergence relations for scattering amplitudes hold in this strongly interacting sector; (b). the sum rules at $t = 0$ are saturated by a minimal set of low-lying states with appropriate quantum numbers. It therefore suggests that a complete description should include all these resonances. These states may lead to distinctive experimental signatures at future colliders.

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Despite the extraordinary success of the Standard Model (SM) in describing particle physics up to the highest energy available today, the mechanism for the electroweak symmetry-breaking remains one of the most prominent mysteries. It has been argued that if there is no light Higgs boson (H) with a mass less than about 1 TeV, the electroweak symmetry-breaking sector would become strongly interacting [1] and new physics must enter. However, it is commonly believed that a heavy Higgs boson may be very broad due to the enhanced couplings among the H and the longitudinal vector bosons, while a ρ -like resonance (denoted by V henceforth) may have a mass close to 2 TeV based on a naive scaled-up version of QCD in various Technicolor models [2]. Those features would make the direct experimental searches for those heavy particles at future colliders difficult and sophisticated acceptance cuts must be applied to enhance the signals over SM backgrounds [3].

In this Letter we would like to argue that the above conventional wisdom on the scalar and vector states may not be true in general. In particular, the separate consideration on each individual low-lying state in a strongly-interacting sector may be incomplete. The well-known example is the strong interactions of hadrons in which the requirement of good high energy behavior of S -matrix elements gives rise to a set of Adler-Weisberger (AW)-type sum rules [4] for forward scattering amplitudes [5]. To satisfy these sum rules, besides the Goldstone bosons π 's, the other states such as ρ, σ, a_1 and ω are required to coexist. We thus propose a scenario for the strongly-interacting electroweak sector which contains a rich spectrum of low-lying states, including the Goldstone bosons (w^\pm, z or equivalently the longitudinal components of the weak bosons W_L in high energy limit), a scalar resonance H , a ρ -like vector resonance V , and other vector resonances such as A_1 and ω_H [6] (analogous to a_1 and ω in low energy hadron physics). The AW-type sum rules and the superconvergence relations are applied to $W_L W_L$, $W_L A_1$ and $W_L V$ scatterings to obtain the relations among the couplings and masses, which are fully expressed by two parameters: a mixing angle and one of the masses. Motivated by this rather coherent picture, we construct an effective chiral Lagrangian containing those resonances with a manifest $SU(2)_L \otimes SU(2)_R$ symmetry

(spontaneously broken down to $SU(2)_V$, the weak-isospin, by the scalar vacuum expectation value v). The ω_H interactions with other states are separately introduced. As a result, many relations among the masses and coupling constants obtained by the AW-type sum rules and superconvergence relations are also preserved in the chiral Lagrangian formalism. The scattering amplitudes obtained from the chiral Lagrangian can thus satisfy the sum rules as required by a proper high energy behavior.

An important consequence is that these low-lying states are in general quite narrow, with widths typically of 100 GeV or less. Even for the scalar state H , its width is significantly smaller than that in the SM due to the coexistence of a vector state V . Also, A_1 and ω_H decay substantially to three W_L 's due to the conservation of angular momentum and isospin. These features may lead to distinctive experimental signatures at the next generation of colliders, such as the Large Hadron Collider (LHC), a TeV e^+e^- Next Linear Collider (NLC) and a possible $e\gamma$ collider. Of equal importance, because of the non-decoupling nature to the SM physics at low energies, those low-lying states may manifest themselves through virtual effects. One may therefore be able to explore the parameter space of the model and to test it by precision electroweak data well before the operation of future colliders.

Before discussing our model in any detail, we recall that, for strong interactions of hadrons, it has been long shown by Gilman and Harari [5] and by Weinberg [7] that *all* Adler-Weisberger-type sum rules can be satisfied with just four meson states, the π , ρ , σ and a_1 . It was recently pointed out by Weinberg [8] that this is a natural result of the algebraic structure of broken symmetry (a “mended symmetry”) $SU(2)\otimes SU(2)$, and these four mesons should actually form a more-or-less complete representation of the group. The mass splitting is accounted for by the non-commutativity between the group generators and the mass matrix. Such an approach is quite successful in relating various couplings and masses in low energy hadron interactions.

We therefore propose that similar sum rules also apply to the strongly-interacting electroweak sector and thus construct a model of possible low-lying states which mimic the algebraic structure in strong interactions. If we ignore the gauge interactions and the isospin

breaking due to mass splitting within fermion multiplets, the parity (P) and the weak isospin (I) are conserved in the electroweak symmetry-breaking sector. If we assign the quantum numbers $I^G(J^P) = 1^-(0^-)$ to the Goldstone bosons (w^\pm, z) and $0^+(0^+)$ to the Higgs boson H , the quantum numbers for the ρ -like resonance V and the axial vector A_1 would be $1^+(1^-)$ and $1^-(1^+)$, respectively.

The algebraic aspects of the symmetry precisely arise from the need for cancellation which ensures a reasonable asymptotic behavior of the S -matrix elements at high energies as required by Regge theory or a renormalizable theory. As a first test of the consistency of the theory, one may consider the process $W_L A_1 \rightarrow W_L A_1$. The Regge theory on the high energy behavior of the forward ($t = 0$) scattering amplitude is best represented in the following dispersion relations: the superconvergence for $I_t = 2$ in the t -channel [5]

$$\int_0^\infty \frac{d\nu}{\nu} \text{Im} \mathcal{T}^{(I_t=2)}(\nu, t=0) = 0, \quad (1)$$

and the usual Adler-Weisberger sum rule [4] for $I_t = 1$ in the t -channel

$$\frac{2}{\pi} \int_0^\infty \frac{d\nu}{\nu^2} \text{Im} \mathcal{T}^{(I_t=1)}(\nu, t=0) = \frac{4}{v^2}. \quad (2)$$

Here ν is the c.m. momentum squared and $v = 246$ GeV. Eq. (1) is derived by assuming the absence of $I = 2$ states (which is true in strong interactions) so that the amplitude satisfies an unsubtracted dispersion relation in ν and its value at threshold vanishes. The existence of H and V states is sufficient to saturate the above sum rules. The aforementioned scenario can be applied to $W_L V$ scattering. One finds that the saturation of forward scattering *requires* the existence of an isosinglet vector boson, the ω -like state, to be denoted by ω_H [$0^-(1^-)$].

We can derive eight Adler-Weisberger type sum rules for our minimal content of the low-lying states in the strongly interacting electroweak sector: two for $W_L A_1$ scattering with contributions from H and V ; two for $W_L W_L$ scattering saturated by H and V ; four for $W_L V$ scattering with contributing states W_L , ω_H and A_1 . There are no sum rules for the scattering of W_L 's on any of the isosinglet states. The complete list of these sum rules can be found in Refs. [5] and [9]. These sum rules yield the following relations:

$$M_V^2 = M_\omega^2 = M_H^2 \tan^2 \psi + M_Z^2 (1 - \tan^2 \psi) \quad ; \quad M_A^2 = \frac{M_V^2}{\sin^2 \psi} - M_Z^2 \cot^2 \psi; \quad (3)$$

$$g_{HWW}^2 = \frac{4}{v^2} \sin^2 \psi \quad ; \quad g_{\omega VW}^2 = \frac{4}{v^2} \quad ; \quad \frac{g_{VWW}^2}{M_V^2} = \frac{1}{v^2} \cos^2 \psi; \quad (4)$$

$$\frac{g_{AHW}^2}{M_A^2} = \frac{1}{v^2} \cos^2 \psi \quad ; \quad g_{AVW}^2 = \frac{16}{v^2} \frac{M_A^2 M_V^2}{(M_A^2 - M_V^2)^2} \sin^2 \psi, \quad (5)$$

where ψ is a free parameter, the mixing angle between W_L and A_1 . None of these states can be eliminated from the sum rules without leading to apparent contradictions. For instance, if the V state is removed from the $W_L A_1$ sum rules by assuming the degeneracy $M_H = M_A$ which corresponds to the “unmixed” case $\psi = 0$, this would then imply a grave violation of the $W_L W_L$ sum rules [5,8,9]. The importance of these results is that the masses and couplings of these low-lying states are completely parameterized by the mass of any one of these states (*e.g.*, M_H) and a mixing angle ψ .

The canonical value of ψ is $\pi/4$ in a QCD-like theory as implied by the KSRF relation [10]. It then follows $M_V \simeq M_A/\sqrt{2}$, the famous relation first derived from the Weinberg sum rule. Note that in this case the scalar width is reduced by about a half compared to that obtained from perturbation theory in the SM. For $\psi = \pi/4$ and some typical values of M_H , we summarize our predictions in Table I. In calculating the ω_H width, we have assumed the V exchange dominance [11]. Indeed, these states are in general quite narrow, with widths typically of 100 GeV or less. (Not listed in Table I are predictions for higher mass values, where *e.g.* for $M_\omega \cong 1.8$ TeV, the two-body decay width is approximately 10 % of the three-body one.) However, we emphasize that the deviation of ψ from $\pi/4$ is quite possible in a theory other than the QCD-like ones (*e.g.* in some Extended Technicolor models [2]) where these sum rules are expected to hold since they are based on a more general ground. Our parametrization allows one to determine the mixing angle from experiments. A preliminary exploration shows that the current precision electroweak data seem to favor certain region of the parameter space with moderate mixing in our model [9].

In order to further study the phenomenology and to gain insight of the underlying physics for the aforementioned model, we construct an effective chiral Lagrangian containing those

low-lying states. Guided by the non-linear σ -model including vector mesons in hadron strong interactions [12], we parameterize the massless Goldstone bosons w_a ($a=1,2,3$) and the vector fields by

$$U = \exp(i\tau_a w_a/v); \quad A_{L(R)}^\mu = \frac{1}{2}(V^\mu \pm A^\mu + \omega_H^\mu), \quad (6)$$

where $V^\mu = \tau_a V_a^\mu$, $A^\mu = \tau_a A_a^\mu$ (with $\text{Tr} \tau_a \tau_b = 2\delta_{ab}$). They transform linearly under $SU(2)_L \otimes SU(2)_R$ as

$$\begin{aligned} A_L^\mu &\rightarrow L A_L^\mu L^\dagger; \quad A_R^\mu \rightarrow R A_R^\mu R^\dagger; \\ U &\rightarrow L U R^\dagger; \quad \omega_H^\mu \rightarrow \omega_H^\mu; \quad H \rightarrow H. \end{aligned} \quad (7)$$

Ignoring the electroweak gauge couplings, the chiral Lagrangian realized in Goldstone mode can then be written as

$$\begin{aligned} \mathcal{L}^{eff} &= \frac{1}{4}v^2\beta \text{Tr}(D^\mu U^\dagger D_\mu U) - \frac{1}{4}\text{Tr}(A_L^{\mu\nu} A_{L\mu\nu} + A_R^{\mu\nu} A_{R\mu\nu}) \\ &+ \frac{1}{2}M_V^2 \text{Tr}(A_{L\mu}^2 + A_{R\mu}^2) + \frac{1}{2}\partial^\mu H \partial_\mu H - \frac{1}{2}M_H^2 H^2 + \frac{1}{2}v \frac{H}{\sqrt{\beta}} \text{Tr}(D^\mu U^\dagger D_\mu U) \\ &- \frac{1}{4}\lambda g_V v H \text{Tr}[i(A_L^\mu U - U A_R^\mu) D_\mu U^\dagger + h.c.] + \mathcal{L}_{\omega_H}, \end{aligned} \quad (8)$$

where

$$D^\mu U = \partial^\mu U - ig_V A_L^\mu U + ig_V U A_R^\mu \quad \text{and} \quad A_{L(R)}^{\mu\nu} = \partial^\mu A_{L(R)}^\nu - \partial^\nu A_{L(R)}^\mu - ig_V [A_{L(R)}^\mu, A_{L(R)}^\nu]$$

with g_V the new strong coupling constant, which we have taken to be the same for A_L and A_R fields since parity is conserved in the new strong interactions. Parameters β [13] and λ are to be fixed by proper normalization and sum rules. The ω_H interactions with other states can be conveniently written as [12]

$$\begin{aligned} \mathcal{L}_{\omega_H} &= -5C g_V \omega_H^\mu \epsilon_{\mu\nu\alpha\beta} \text{Tr}[L^\nu L^\alpha L^\beta] - \frac{15}{4}C g_V^2 \omega_H^{\mu\nu} \epsilon_{\mu\nu\alpha\beta} \text{Tr}[V^\alpha (R^\beta - L^\beta) \\ &+ A^\alpha (R^\beta + L^\beta) + V^\alpha A^\beta - \frac{1}{2}(V^\alpha - A^\alpha)U^\dagger (V^\beta + A^\beta)U]. \end{aligned} \quad (9)$$

Here C is an arbitrary constant to be fixed by the sum rules and $L_\mu = U^\dagger \partial_\mu U = -U^\dagger R_\mu U$. (In QCD, $C = N_c/240\pi^2$.)

Diagonalizing the quadratic crossing term in \vec{A}_μ and $\tilde{D}_\mu \vec{w} (\equiv \partial_\mu \vec{w} + g_V \vec{V}_\mu \times \vec{w})$ in Eq. (8), one obtains

$$\beta = M_V^2 / (M_V^2 - g_V^2 v^2), \quad M_A^2 = M_V^2 + \beta g_V^2 v^2. \quad (10)$$

Upon defining

$$\beta \equiv \frac{1}{\sin^2 \psi}, \quad \lambda \equiv 2 \left(\frac{1}{\sin \psi} - \sin \psi \right), \quad M_H^2 \equiv \beta g_V^2 v^2, \quad (11)$$

the mass relations in Eq. (3) and those for g_{HWW} and g_{VWW} in Eq. (4) immediately follow. The sum rule relation for g_{AHW} can be easily obtained as well¹. The second term of \mathcal{L}_{ω_H} in Eq. (8) gives the sum rule for $g_{\omega VW}$ if we choose

$$C \equiv \frac{1}{15g_V^2}. \quad (12)$$

It is very interesting to note that with such a simple effective Lagrangian of Eq. (8), many relations among masses and couplings in Eqs. (3)-(5) obtained by strong scattering sum rules are preserved. The only exception is the last one in Eq. (5) involving g_{AVW} which will need some more careful treatment. We shall address this point in a later work [9].

Unlike most of the previous studies [3,14], which makes no reference to the dynamical relationship and detail cancellation among the proposed resonances, our scenario is based on a consistent parametrization of low-lying states with certain internal symmetry, obeying strong interaction sum rules as required by a proper high energy behavior. These low-lying narrow states could lead to distinctive experimental signatures at future colliders. Even without direct couplings to fermions, the vector states V , A_1 and ω_H may be copiously produced at the LHC and a TeV e^+e^- NLC via a mixing with the transversely polarized vector bosons γ, Z or W^\pm . A_1 and ω_H may also be produced via the fusion processes $W_L \gamma \rightarrow A_1, \omega_H$, most conveniently at an $e\gamma$ collider. The dominant three-body decays of A_1

¹To derive the proper Feynman rule for the AHW interaction, an integration by parts was used to get a term like $\vec{A}_\mu \cdot \vec{\pi} \partial^\mu H$.

and ω_H would provide distinctive signature from the two-body decays of V . For instance at the LHC for $\sqrt{s} = 14$ TeV, the production cross section for a 900 GeV H is about 0.2 pb; while that for the vector states is of order 1 pb. With the designed annual integrated luminosity of 100 fb^{-1} , one would expect $10^4 - 10^5$ $2W_L$ or $3W_L$ final states. With the narrow widths for the vector states, as well as the significantly reduced width for H , the experimental identification for these low-lying states at the LHC should be quite promising. Comprehensive phenomenological studies of these states will be reported elsewhere [9].

To summarize, we propose that in a strongly-interacting electroweak sector, to satisfy the Adler-Weisberger-type sum rules and the superconvergence relations, there exists a rich spectrum, including the Goldstone bosons w_a , a scalar H , a vector V , an axial vector A_1 and an isospin-singlet vector ω_H , obeying certain algebraic structure. By applying the AW-type sum rules, relations among masses and couplings of those states are established and they are fully expressed by two parameters: a mixing angle ψ and one of the masses (say, M_H). It is found that in general those states are fairly narrow, due to the necessary coexistence of the states. Our model yields distinctive predictions and experimental signatures. Motivated by this model, we construct an effective chiral Lagrangian for these states with a manifest $SU(2)_L \otimes SU(2)_R$ symmetry. Resultant mass and coupling constant relations satisfy almost all the sum rules. This effective Lagrangian can be easily applied to future phenomenological studies. It is crucial to test these ideas at the forthcoming LHC and/or NLC facilities and to explore the fundamental mechanism for the electroweak symmetry-breaking.

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TABLES

TABLE I. Predictions on the masses and widths (in units of GeV) of A_1 , V and ω_H states for the mixing angle $\psi = \pi/4$ and some input values of M_H .

M_H	800	1000	1200
$\Gamma(H \rightarrow W_L W_L)$	120	238	416
M_A	1131	1414	1697
$\Gamma(A_1 \rightarrow V W_L)$	31.5	67	121
$\Gamma(A_1 \rightarrow H W_L)$	8.5	17.6	31.3
M_V	800	1000	1200
$\Gamma(V \rightarrow W_L W_L)$	26.1	52.4	92
M_ω	800	1000	1200
$g_{\omega V W}$ (GeV $^{-1}$)	8.1×10^{-3}	8.1×10^{-3}	8.1×10^{-3}
$\Gamma(\omega_H \rightarrow Z_L W_T^3)$	2.2	2.8	3.3
$\Gamma(\omega_H \rightarrow Z_L W_L^+ W_L^-)$	1.5	4.1	8.9

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